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Seismic inversion for reservoir properties combining statistical rock physics and geostatistics: A review

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ABSTRACT

There are various approaches for quantitative estimation of reservoir properties from seismic inversion. A general Bayesian formulation for the inverse problem can be implemented in two different work flows. In the sequential approach, first seismic data are inverted, deterministically or stochastically, into elastic properties; then rock-physics models transform those elastic properties to the reservoir property of interest. The joint or simultaneous work flow accounts for the elastic parameters and the reservoir properties, often in a Bayesian formulation, guaranteeing consistency between the elastic and reservoir properties. Rock physics plays the important role of linking elastic parameters such as impedances and velocities to reservoir properties of interest such as lithologies, porosity, and pore fluids. Geostatistical methods help add constraints of spatial correlation, conditioning to different kinds of data and incorporating subseismic scales of heterogeneities.

INTRODUCTION

This paper reviews seismic inversion schemes (deterministic and stochastic) that incorporate rock-physics information and geostatistical models of spatial continuity. We focus on techniques that go beyond inverting for the elastic parameters (e.g., impedances, elastic moduli) and try to infer reservoir properties of interest, such as lithologies, porosities, and fluid saturations. Doyen's book (2007) is an excellent, easy-to-read summary of many of the methods described.

The transformation of any geophysical data into physical properties of the earth such as elastic or electrical parameters can be posed as an inverse problem. General inverse theory is a mathematically rich discipline, and many excellent books on geophysical inverse theory exist (e.g., Menke, 1984; Tarantola, 1987; Parker, 1994; Sen and Stoffa, 1995; Oliver et al., 2008). However, the goal of using geophysical methods for reservoir characterization usually goes beyond estimating the physical quantities to which a remote-sensing experiment responds. Rather, the final goal usually is to infer reservoir properties that include characteristics of the rocks (lithology, fluid, porosity) and the regime of physical conditions (pressure, temperature) to which they are subjected.

Seismic-reflection data are used in reservoir characterization not only for obtaining a geometric description of the main subsurface structures but also for estimating properties such as lithologies and fluids. However, transforming seismic data to reservoir properties is an inverse problem with a nonunique solution. Even for noise-free data, the limited frequency of recorded seismic waves makes the solution nonunique. The inversion of seismic data for reservoir properties is more complicated in practice because of (1) the ever-present noise in the data, (2) the forward-modeling simplifications needed to obtain solutions in a reasonable time, and (3) the uncertainties in well-to-seismic ties (depth-to-time conversion), in estimating a representative wavelet, and in the links between reservoir and elastic properties.

We focus our review on methods of seismic inversion for reservoir characterization that incorporate geology, rock-physics or petrophysical knowledge, and geostatistics. A combination of elastic-property estimates from seismic inversion and rock physics or petrophysics for predicting reservoir properties is a key and classical procedure in reservoir characterization. Recent developments include rock-physics relations as constraints to the inversion. Geostatistics includes the constraints imposed by spatial correlation, represented by variograms, Markov random-field models, or multipoint spatial statistics captured in training images. Geostatistical methods simulate small-scale variability not captured in seismic data because of limited resolution. Geostatistical models can be used at the beginning of the stochastic inversion to provide geologically consistent

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prior models or after extracting and classifying seismic attributes to impart realistic geologic patterns and spatial correlation.

Before describing approaches in quantitative seismic interpretation, we discuss some of the important reasons for quantifying uncertainty of the interpretation.

WHY QUANTIFY UNCERTAINTY?

Subsurface-property estimation from remote geophysical measurements is always subject to uncertainty because of many inevitable difficulties and ambiguities in data acquisition, processing, and interpretation. Most interpretation techniques give us some optimal estimate of the quantity of interest. Obtaining the uncertainty of that interpretation usually requires further work and hence comes at an extra cost. So what extra benefits do we receive? Why care about quantifying uncertainty? Indeed, why quantify uncertainty at all?

Uncertainty in assessing reservoir properties from seismic data comes from several sources. One source of uncertainty is measurement errors in the seismic and well-log data. In the case of seismic information, the original data are processed to form seismic images of the subsurface. Similarly, the original well measurements are processed to calculate a series of medium properties. In both cases, the data-processing stage involves uncertainty. Seismic modeling is based on an approximate wave-propagation model for the medium, such as the isotropic elastic model commonly used for inversion. In addition, uncertainty is associated with the rock-physics transforms from elastic properties to reservoir properties and scale issues. Houck (2002) shows a formulation to account for geologic, interpretation, and measurement uncertainties in a Bayesian framework to interpret seismic amplitude-variation-with-offset (AVO) data for reservoir properties. Quantitative seismic-interpretation schemes attempt to account for different sources of uncertainties (or at least parts of them) in different ways.

One fundamental reason for quantifying uncertainty stems from our accountability as responsible scientists. We know that models are approximate, data have errors, and rock properties are variable. So as responsible scientists, we appropriately report error bars along with interpretation results. Error bars lend credibility. A more practical reason for understanding uncertainty is for risk analysis and optimal decision-making. Quantifying uncertainty helps us to estimate our risk better and possibly to take steps to protect ourselves from that risk. Uncertainty assessments are also useful in data integration and in estimating the value of additional information for reducing uncertainty. Complex interpretation processes such as reservoir characterization usually require integrating different types of data from different sources. Understanding the uncertainties associated with the different data sets helps us to assign proper weights (and discard unreliable data) before we combine them in the interpretational model. Additional data (e.g., S-wave data in addition to P-wave seismic data) may help to clarify ambiguities and reduce uncertainty — but not always. Estimating the value of additional information requires quantitative assessment of uncertainty.

SEISMIC DATA TO RESERVOIR PROPERTIES: POSING THE PROBLEM

Any inversion problem can be posed as a Bayesian inference problem, i.e., update the prior knowledge, accounting for observations (e.g., Tarantola, 1987, 2005; Duijndam, 1988a, 1988b; Ulrych et al., 2001). It can be expressed as Posterior = Constant × Prior × Likelihood. Using symbols, it can be expressed as

$$\sigma_{\text{post}}(\mathbf{m}) = c\rho_{\text{prior}}(\mathbf{m})\rho_{\text{data}}(\mathbf{d}_{\text{obs}} - \mathbf{g}(\mathbf{m})), \tag{1}$$

where $\sigma_{\text{post}}(\mathbf{m})$ is the posterior probability density and $\rho_{\text{prior}}(\mathbf{m})$ is the a priori probability density. The value $\rho_{\text{data}}(\mathbf{d}_{\text{obs}} - \mathbf{g}(\mathbf{m}))$ is the data-likelihood function; it depends on the observations $\mathbf{d}_{\text{obs}}$ and their uncertainties, the forward-modeling operator $\mathbf{g}$ that maps the model space into the data space, and the modeling uncertainties. In equation 1, $\mathbf{m}$ specifies the earth model parameter configuration, and c is a constant for normalizing the posterior probability density. The forward operator may be a simple function with an analytic form, a matrix operator, or, more generally, an operator or computational algorithm with no simple analytic expression.

The general formulation in equation 1 accepts different types of solution approaches. Two major lines are (1) to search the maximum of the posterior probability density or (2) to produce samples from the posterior probability density. The first category is commonly called optimization, error minimization, or the deterministic approach. The second category is known as the stochastic, Monte Carlo, or sampling approach. The sampling approach is more general because it assesses the marginal posterior probability density and not just the maximum probability. However, the two approaches conduce to the same maximum probability estimate if the same information, data, and physics are used.

The solutions of an inverse problem are the set of earth-model configurations that, when forward modeled into synthetic data, match the real data within some tolerance. Specifically, in seismic inversion, we invert for models of elastic properties: P-wave impedance, S-wave impedance (or velocities), and density. However, for reservoir characterization, these realizations of elastic parameters must be transformed to useful reservoir properties such as lithologies, porosities, and saturations. When the model parameterization is directly in terms of reservoir properties (lithofluid classes, net to gross, porosity), then the likelihood includes the rock-physics model relating reservoir properties to elastic properties as well as the wave-propagation model relating elastic properties to seismic data. Spatial correlation of the model parameters may be incorporated into the prior model, though not all of the published Bayesian inversion methods do so.

The formulation for the joint seismic and petrophysical inversion requires a partition of the model space in reservoir and elastic medium parameters $\mathbf{m} = (\mathbf{m}_{\text{res}}, \mathbf{m}_{\text{elas}})$ and decomposing the prior information factor, in equation 1, according to the chain rule (e.g., Bosch, 1999, 2004):

$$\rho_{\text{prior}}(\mathbf{m}_{\text{res}}, \mathbf{m}_{\text{elas}}) = \rho_{\text{petro}}(\mathbf{m}_{\text{elas}} | \mathbf{m}_{\text{res}})\rho_{\text{prior}}(\mathbf{m}_{\text{res}}), \tag{2}$$

where $\rho_{\text{prior}}(\mathbf{m}_{\text{res}})$ is the prior probability density for the reservoir parameters and $\rho_{\text{petro}}(\mathbf{m}_{\text{elas}} | \mathbf{m}_{\text{res}})$ is a conditional probability for the elastic parameters that summarizes the rock-physics or petrophysical information on reservoir- and elastic-property relationships. Similar formulations for the prior density in the joint petrophysical and seismic inversion are described by Eidsvik et al. (2004), Gunning and Glinsky (2004), Larsen et al. (2006), Bosch et al. (2007), Spikes et al. (2007), Buland et al. (2008), González et al. (2008), and Bosch et al. (2009b). Grana and Della Rossa (2010) use Gaussian mixture models in a Bayesian framework, combining statistical rock physics with seismic inversion to obtain a probabilistic estimation of petrophysical properties.

The formulation of the posterior probability density for the joint petrophysical and seismic inversion can be written by inserting the expanded prior information in expression 1 and modeling the conditional density (e.g., Bosch, 2004):

$$\sigma_{\text{post}}(\mathbf{m}_{\text{res}},\mathbf{m}_{\text{elas}}) = c\rho_{\text{petro}}(\mathbf{m}_{\text{elas}} - \mathbf{f}(\mathbf{m}_{\text{res}}))\rho_{\text{data}}(\mathbf{d}_{\text{obs}} - \mathbf{g}(\mathbf{m}_{\text{elas}}))\rho_{\text{prior}}(\mathbf{m}_{\text{res}}), \quad (3)$$

where the petrophysical conditional density is $\rho_{\text{petro}}(\mathbf{m}_{\text{elas}} - \mathbf{f}(\mathbf{m}_{\text{res}}))$, with $\mathbf{f}$ being the petrophysical forward function that maps the reservoir model parameters to the elastic model parameters. In general, the petrophysical factor accounts for the deviations of the elastic parameters from their expected petrophysical transformed values, uncertainties of the petrophysical forward function, and spatial variability of the deviations.

Many different work flows combine seismic inversion for elastic properties, geostatistics, and rock-physics models to predict reservoir properties. The work flows may be grouped in two categories, depending on whether the combination is stepwise or unified in a single formulation. The sequential or cascaded approach (Dubrule, 2003; Doyen, 2007) consists of first inverting the seismic data for elastic properties and then using rock-physics models and a statistical classification scheme to transform the elastic properties to reservoir properties, which may then be used as soft data to constrain geostatistical simulations of reservoir properties. The seismic inversion for elastic properties may be a deterministic gradient-based inversion or a stochastic inversion. In contrast, the joint or simultaneous work flow accounts for the elastic parameters and the reservoir properties together, often in a Bayesian formulation, guaranteeing consistency between the elastic and reservoir properties.

The justification of a unified formulation for the seismic and petrophysical inversions can be considered in the issues of exactitude, precision, and quantification of the uncertainties. The exactitude (absence of bias) of parameter estimates (mean, maximum probability, median) is equivalent between the cascaded and joint work flows if the relationship between elastic and reservoir parameters is linear; for other cases, the unified approach improves the exactitude of estimates (Bosch, 2004). The unified approach also provides a formulation for calculating combined uncertainties (petrophysical dispersion and overlapping of distributions, seismic modeling, and observation errors), particularly in Monte Carlo solutions. However, the difference between work flows remains methodological, whereas the crucial issue is the combination of quality seismic, petrophysical, and geostatistical information. Results with different work flows may be very close and differences may be negligible when the same data, prior information, physics, parameterization, and spatial models are used. Figure 1 shows three different work-flow schematics.

SEISMIC INVERSION

Although our focus is on methods for assessing reservoir properties, in this section we briefly describe some of the traditional methods for inverting for elastic properties from seismic data because elastic-property estimates are often an intermediate step in inferring reservoir properties in seismic reservoir characterization. The band-limited nature of the seismic data, together with observational and modeling uncertainties, gives rise to the inherent nonuniqueness of the seismic-inversion-for-elastic-properties problem. In other words, many configuration combinations of elastic earth models fit the data equally well.

The inversion of seismic data for elastic properties can be posed as a deterministic problem or as a stochastic problem, i.e., model random parameters characterized by probability densities. Russell (1988) summarizes two of the widely used deterministic or optimization-based methods of seismic inversion for elastic properties: sparse-spike techniques and model-based inversion. Sparse-spike techniques deconvolve the seismic trace under sparseness assumptions of the reflectivity series, an idea initially proposed by Oldenburg et al. (1983). First, reflectivities are obtained; then, impedances are computed, including the missing low frequencies, usually from well data, seismic-velocity analysis, or a kriging estimate of the low-frequency trend. In model-based methods (Russell and Hampson, 1991), starting from a given initial model, the inversion algorithm perturbs the model until some minimization criteria are satisfied. The objective function, or the function to be minimized, is usually a difference between the observed and modeled data. However, additional regularization terms are usually included in the objective function to restrict possible solutions to those that satisfy additional criteria, such as a fixed mean layer thickness, smoothness, or a condition of lateral continuity.

Gradient-based methods attempt to solve nonlinear minimization problems by linearizing around an initial solution. Iterative linear steps are taken to update the current model based on gradient information. The iteration stops when errors in the updates are below some tolerance. Gradient methods such as steepest descent and conjugate gradients or Hessian methods such as Newton's can be used to minimize the objective function; they are sensitive to the choice of the starting point and can easily get trapped in local mimima.

Cooke and Schneider (1983) describe generalized linear least-squares inversion for impedances from normal-incidence seismic

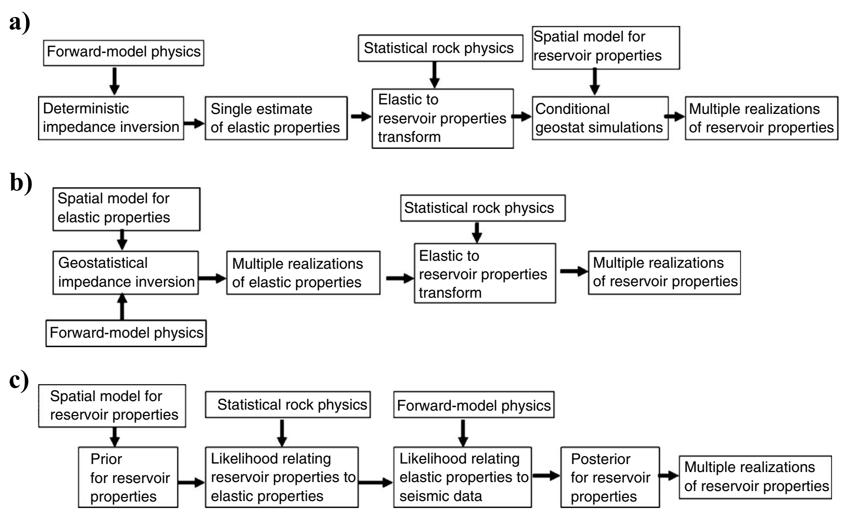

Figure 1. Schematic of possible work flows in seismic inversion for reservoir properties leading to multiple realizations of reservoir properties conditioned to seismic data, the rock-physics model, and the model of spatial continuity imposed by geology. (a, b) Two variations of the sequential (or cascaded) work flow. (c) Simultaneous Bayesian inversion work flow. Data come in at various stages, including seismic data for inversion, well-log data, cores and thin sections when available, and geologic data from outcrops.

traces, but Mora (1987) presents a preconditioned conjugate-gradient algorithm for nonlinear least-squares inversion of multioffset full-waveform seismic data. Sen and Stoffa (1991) and Ma (2002) use simulated annealing to tackle the nonlinear optimization problem. The monograph by Sen and Stoffa (1995) describes many of the minimization techniques used in seismic inversion, both descent methods, and global optimization techniques such as simulated annealing and genetic algorithms. Mallick (1995, 1999) describes some practical aspects of using genetic algorithms for prestack waveform inversion.

The Bayesian formulation presented in equation 1 can be very difficult to solve in a closed analytical form. Under appropriate assumptions, equation 1 leads to particular types of problems with well-established methods for finding the solutions. For example, as Tarantola (1987) shows, by assuming Gaussian distributions for the prior probability density and for the errors in the data and by using a linear forward-model operator, equation 1 yields a posterior probability density that is also Gaussian. In this situation, the mean and covariance of the posterior probability density are given by the solution of a least-squares problem; that is, means and covariances, and hence the complete posterior distribution, are defined analytically. A less strong assumption about the forward-model operator is that it is smooth enough to be approximated locally by a multivariate linear or quadratic function. In this approach (also in Tarantola, 1987), common nonlinear least-squares methods such as Newton's can be used iteratively to search for the maximum posterior probability model configuration at the closest mode. However, convergence to the global maximum is not ensured unless the posterior probability density is monomodal. A posterior covariance can be calculated at the local maximum, which only accounts for the uncertainty of the corresponding mode under the local quadratic approximation.

Diverse methods such as simulated annealing can be used to overcome the problem of multimodality. However, without assumptions such as the ones just mentioned, the general solution to the inverse problem consists of enough samples from the posterior probability density that can be obtained using Monte Carlo (Tarantola, 1987) or sequential simulation methods. Lörtzer and Berkhout (1992) perform a linearized Bayesian inversion of seismic amplitudes based on a single-interface theory. Gouveia and Scales (1998) define a Bayesian nonlinear model and estimate the maximum a posteriori elastic parameters. Buland and Omre (2003) and Buland et al. (2003) have developed a linearized Bayesian AVO inversion method, accounting for the wavelet using a convolution model. They obtain explicit analytical expressions for the posterior density of the elastic parameters, providing a computationally fast method. Grana and Della Rossa (2010) use Gaussian-mixture models, thus avoiding the restrictions of a Gaussian assumption and taking advantage of analytical expressions available for conditional distributions of Gaussian-mixture models.

ROCK PHYSICS

Rock physics is included in quantitative seismic interpretation as a cascaded step after seismic inversion or within a joint seismic/petrophysical formulation, linking elastic parameters and reservoir properties. Theoretical and empirical rock-physics models typically describe the behavior of elastic moduli as a function of factors such as mineralogy, porosity, pore type, pore fluids, clay content, sorting, cementation, and stress. We do not review specific rock-physics models, so the reader is referred to review papers such as Berryman (1995) and Avseth et al. (2010), the collection of papers edited by Wang and Nur (1992, 2000), and rock-physics books (e.g., Guéguen and Palciauskas, 1994; Schön, 1996; Mavko et al., 2009).

Ideally, the values of the elastic properties derived from inverting seismic data are assigned to a specific depth or time zone; therefore, the transformation from elastic to reservoir properties may be done point by point. However, application of rock-physics models derived at the log or core scale to band-limited seismic inversion results can be problematic because the inversion results represent seismic-scale aggregate lithologies (Doyen, 2007). This problem can be treated with appropriate scale transforms for the reservoir and elastic parameters.

Calibration of the rock-physics model using log data and seismic synthetic modeling is necessary. With enough training data, variations of linear or nonlinear regression, geostatistical, and neural-network techniques can be used to empirically convert elastic properties to reservoir properties without understanding the physical bases of the transformations. However, applying any statistical correlation without regard to the underlying physics can easily yield erroneous results (e.g., Hirsche et al., 1998). Furthermore, it is very difficult to support predictions of reservoir properties that are not sampled by well logs or training data, a common situation in frontier exploration. Rigorously, the interpretation is limited to the training data used to derive the statistical correlation. Here, rock-physics models play a critical role in deriving correlations between elastic and reservoir properties for scenarios not sampled in the training data.

Including rock physics not only validates the transformation to reservoir properties but also makes it possible to enhance well-log or training data based on geologic processes (e.g., Avseth et al., 2005). In particular, Mukerji et al. (1998), Mukerji et al. (2001a), and Mukerji et al. (2001b) formally introduce statistical rock-physics methods as a way to combine rock physics, information theory, and geostatistics in quantitative reservoir characterization. Earlier pioneering workers who combine rock-physics models with geostatistical algorithms to infer reservoir properties include Doyen (1988), Lucet and Mavko (1991), and Doyen and Guidish (1992).

Statistical rock physics combines theoretical and empirical rock-physics models with statistical pattern-recognition techniques to interpret elastic properties obtained from seismic inversion and to quantify interpretation uncertainty. Statistical rock physics is also useful for identifying additional information that may help reduce interpretation uncertainties. The statistical rock-physics methodology can be divided into four broad steps.

First, well-log data are analyzed to obtain facies definition. This is done after appropriate corrections, including fluid substitution and shear-velocity estimation when required. Knowledge of background geology as well as core and thin-section information (when available) also play an important part in this step. For each facies, basic rock-physics relations, such as velocity/porosity and P-/S-wave velocity V_P-V_S, are defined and modeled using appropriate theoretical or empirical models. We use the term *facies* for categorical groups — not necessarily only by lithology type but also by some property or collection of properties, as, for example, a combination of lithology and pore fluids. Brine sands and oil sands are considered two different facies or categories. This general procedure also applies for characterizing continuous reservoir properties such as porosity and net to gross.

This first step is followed by Monte Carlo simulation of seismic rock properties (V_P, V_S, and density) and computations of the facies-dependent statistical probability density functions (PDFs) for seismic attributes of interest (Figure 2). A key feature is the use of rock-physics modeling to extend the PDFs to situations that are of interest but were not encountered in the wells (e.g., different fluid saturations, presence of fractures, different levels of diagenesis or cementation, different depths). For example, in wells with missing V_S, V_S prediction must be conducted using V_P-V_S relations appropriate for the facies; Gassmann's equations can be used for fluid substitution; and lithology substitution can be done using various rock models of cementation, sorting, and clay content (e.g., Avseth et al., 2005; Mavko et al., 2009). The extended PDFs are the derived distributions, conditioned to each facies class: the probability of attributes given the facies, or $P(\text{attributes}|\text{facies})$. Using the derived PDFs of seismic attributes, feasibility evaluations are made about which set of seismic attributes contains the most information for the problem. Discriminating lithologies may require a different set of attributes than, say, discriminating fractured versus unfractured reservoir zones. An evaluation of the well-log-based seismic-attribute PDFs can guide the choice of attributes to be extracted from the seismic data.

For the third step in the work flow, the elastic properties (or related attributes) from seismic inversion are used, in a statistical classification technique, to classify the voxels within the seismic-attribute cube. Calibrating the attributes with the probability distributions defined at well locations allows us to obtain a measure of the probability of occurrence of each facies. This can be done using Bayes' theorem to get the posterior probability of facies given the attributes, or $P(\text{facies}|\text{attributes})$, which is proportional to the product of the likelihood $P(\text{attributes}|\text{facies})$ obtained in step 2 and the prior probability of facies, or $P(\text{facies})$, obtained from geology. Various standard statistical validation tests can be performed to obtain a measure of the classification success. Neural networks can also be used for this classification estimation of the probability of facies given attributes (e.g., Caers and Ma, 2002). Figure 3 shows an example of using Bayesian classification to predict posterior probabilities of oil sands. Near- and far-offset impedances obtained from a deterministic inversion were used as input attributes.

The fourth step in the complete formulation of statistical rock physics (Mukerji et al., 2001a; Avseth et al., 2005) includes applying geostatistical stochastic simulation for imposing spatial correlation. The probabilities obtained in step 3 from classifying seismic attributes depend on the local voxel values of the seismic attributes and are not conditioned to the neighboring spatially correlated values. Hence, this final geostatistical step may be used to update the seismically derived probabilities by taking into account geologically reasonable spatial correlation and by conditioning to the facies and fluids observed at the well locations.

The general work flow is applicable to categorical variables such as lithofacies or for characterizing continuous properties such as porosity and net to gross. The work of Saltzer et al. (2005), Bachrach (2006), and Sengupta and Bachrach (2007) are examples of the sequential or cascaded work flow, using deterministic inversion followed by statistical rock-physics methods to infer reservoir properties such as shale fraction, porosity, and saturations. Similarly, Sams and Saussus (2007) use a deterministic impedance inversion followed by Bayesian rock-physics transforms. Grana and Della Rossa (2010) use a linearized Bayesian inversion to estimate P- and S-wave impedances, followed by statistical rock-physics modeling in a Bayesian framework to calculate the conditional probabilities of petrophysical variables (porosity, water saturation, and clay content) and lithofluid classes (oil-sand, water-sand, and shale) conditioned to the seismic impedances.

These procedures apply in a similar manner for the joint seismic and petrophysical inversion work flow, with a variation in step 3. Instead of a cascaded estimate of elastic and reservoir parameters, they are jointly estimated, honoring within uncertainties the seismic reflection data and the petrophysical relationships. Examples of a joint approach include work by Leguijt (2001, 2009), Bosch (2004), Eidsvik et al. (2004), Gunning and Glinsky (2004), Larsen et al. (2006), Bosch et al. (2007), Spikes et al. (2007), Buland et al. (2008), González et al. (2008), Bosch et al. (2009a), and Bosch et al. (2009b).

Figures 4 and 5 illustrate the joint approach for estimating total porosity and acoustic impedance in the setting of clastic and carbonate sequences at a heavy-oil producing field (from Bosch et al., 2009a). Figure 4 shows total porosity versus acoustic impedance crossplots, derived from well logs and upscaled to the seismic resolution. As shown in the figure, the acoustic impedance is well correlated to the total porosity in this area. A petrophysical model calibrated to the data is also shown, which is used to define the conditional petrophysical density in the expression 3. Thanks to this petrophysical coupling, the porosity and impedance are estimated jointly with the petrophysical seismic inversion technique, as shown in Figure 5.

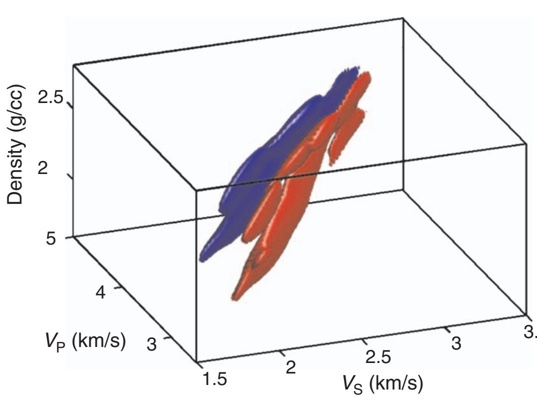

Figure 2. Isosurfaces of trivariate nonparametric PDFs for V_P, V_S, and density for brine sands (blue) and gas sands (red). From Avseth et al., 2005.

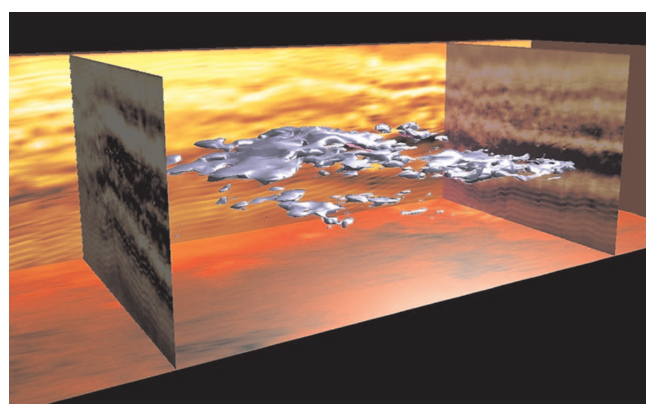

Figure 3. Result of Bayesian classification of near- and far-offset impedances using a statistical rock-physics work flow. Isoprobability surface shows 75% probability of oil-sand occurrence in a North Sea reservoir. Vertical extent is about 100 m; lateral extent is 12 km along the long dimension. From Avseth et al., 2005.

In this area, shale layers are characterized by low acoustic impedance (and corresponding large total porosity), which can be identified from the impedance and porosity sections. The high-acoustic-impedance (and small total porosity) layers correspond to carbonate and sand, which cannot be discriminated in this area by their P-wave impedance alone. Major interpreted strata correlating with well information are indicated in the figure.

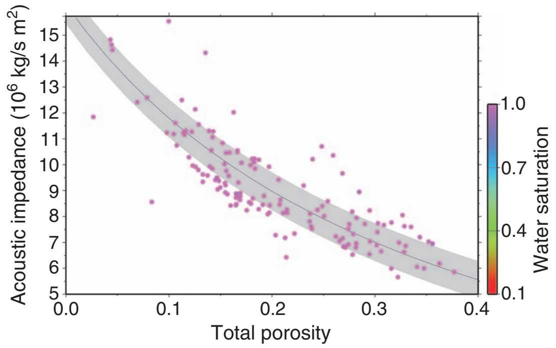

Figure 4. Crossplot of well-log properties rescaled at seismic resolution: acoustic impedance versus total porosity. Colors indicate water saturation. The continuous black line shows the petrophysical transform calibrated to the well-log data. The gray band indicates plus and minus one standard acoustic impedance deviation from the transform, as used in the petrophysical statistical model. Adapted from Bosch et al., 2009b.

Depending on the stage of the reservoir's exploration, development, and production cycles, the steps outlined above may be modified. Not all of the steps may be carried out in the initial exploration stages, where there is little or no well data. In the exploration stage, the PDFs of just a few basic facies categories (say, shale, oil sand, brine sand) can be estimated from wells and a quick classification done using seismic attributes derived at a few locations, e.g., a few AVO intercepts and gradients derived from a handful of common-depth-point (CDP) gathers. In some cases at the early exploration stage, there may be no wells, and the PDFs of rock properties may be based on rock-physics models for analogous data from regions of similar geologic history. In the development stage, based on more extensive well data, additional facies categories may be defined (e.g., shale, unconsolidated sand, cemented sand). Seismic attributes extracted after careful inversions over a full 3D volume may be used in the classification.

ADDING SPATIAL CONSTRAINTS

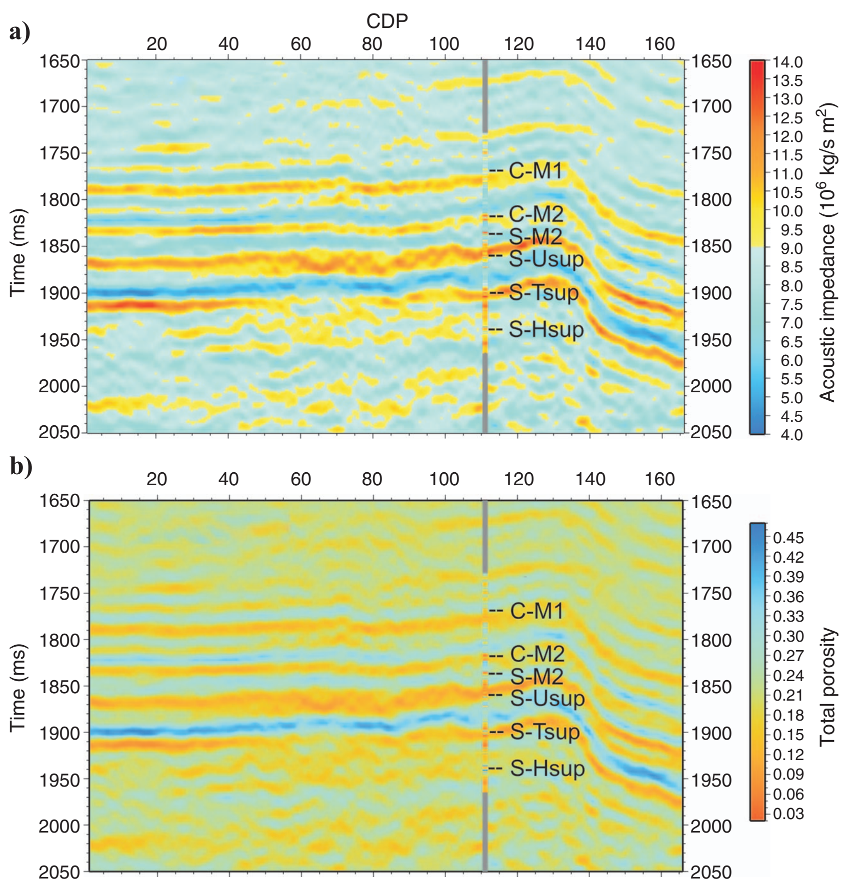

Figure 5. Results of (a) acoustic impedance and (b) total porosity sections resulting from the petrophysical inversion of the seismic data shown in Figure 4 with no well-log conditioning. The acoustic impedance and total porosity calculated from the well-log data are superimposed for comparison with the inversion results. Tops for major carbonate and sand layers are shown. Names beginning with C indicate carbonate layer; those with S indicate sand. Adapted from Bosch et al., 2009b.

Reservoir characterization requires integrating different kinds of data while incorporating the spatial correlation of reservoir heterogeneities. One objective is to give priority to solutions or configurations of the model parameters consistent with particular spatial correlations and crosscorrelations, commonly characterized from geologic and well-log information. A second objective is to combine information from the well-log-based properties with the information of the seismic-derived property estimates so that the resulting field honors the well-log data. A third objective is to incorporate subseismic-scale heterogeneities with spatial correlations consistent with well and geologic information.

Conditioning the inversion to well-log data can be done directly in the seismic scale by transforming well-log-derived properties to the seismic model support with the appropriate change of scale expressions. In this case, however, there is no improvement of vertical resolution because of the combination of well and seismic information. The typical geostatistical approach consists of defining a property model at the subseismic scale, where properties are to be estimated or jointly simulated, and conditioning to the seismic and well-log data (see Haas and Dubrule [1994]; González et al. [2008]; Bosch et al. [2009b]). In this case, an improvement of vertical resolution in the simulations is achieved at distances around the range of correlation with the wells. However, this approach requires that the forward model include the appropriate upscaling — directly, using a full-physics wave-propagation model, or approximately, using a dual-scale model consisting of an upscaling function followed by a simplified wave-propagation model. Figure 6 describes the model parameters and relations for different types of geostatistical seismic inversion conditioned to well-log data.

Spatial variability is typically modeled using various geostatistical methods. Some references on general geostatistical methods are Deutsch and Journel (1998), Dubrule (2003), and Caers (2005). In the sequential work flow of seismic inversion for reservoir properties, geostatistical methods can be used in step 1 (seismic inversion for reservoir properties), in step 2 (transformation of elastic properties to reservoir properties), or in a final step after estimating elastic and reservoir properties. In the latter approach, the localized well information and the 3D properties estimated from the seismic data are combined for estimating the property fields at subseismic scale. Cokriging is a classic geostatistical method that has been used for this purpose. However, kriging techniques are for point estimation and do not honor true spatial variability. Geostatistical cosimulation is essential to imprint true spatial variability to the property fields. Examples are given by Doyen (1988), Lucet and Mavko (1991), Doyen and Guidish (1992), Zhu and Journel (1993), and Mukerji et al. (1998). Mukerji et al. (2001a) use near- and far-offset seismic stacks to invert deterministically for the near- and far-offset impedances. These are input into a Bayesian classification scheme using statistical rock-physics models to obtain probability maps of different lithofluid classes. These probabilities are then updated using a geostatistical Markov-Bayes indicator cosimulation to get the posterior probabilities and multiple realizations of lithofluid categories.

However, in geostatistical inversion for elastic properties (Bortoli et al., 1993; Haas and Dubrule, 1994), geostatistical simulations are more closely integrated with the seismic inversion at the initial stage itself. The original methodology of Bortoli et al. (1993) and of Haas and Dubrule (1994) consists of local trace-by-trace optimization combined with sequential geostatistical sampling based on the horizontal and vertical variogram (Rowbotham et al., 1998). The variogram statistically quantifies the spatial correlation of the impedance. Each trace location is visited in a random path. At each location, a number of possible vertical seismic impedance logs are simulated using sequential Gaussian simulation (Deutsch and Journel, 1998). The simulation is constrained by the existing impedances at the well locations and by the vertical and horizontal variograms. The synthetic seismograms computed from the simulated impedance logs using a 1D convolution model are compared with the actual seismic data. The simulated log that gives the best fit to the seismic data is retained and used as a constraint for simulating vertical logs at the next random location. The seismic data constrain the inversion within the seismic bandwidth, but the higher spatial frequencies are stochastically constrained by the variograms obtained from well logs and the hard data at the wells.

Debeye et al. (1996) present a stochastic inversion scheme that is equivalent to the Haas and Dubrule method with the addition of a simulated annealing component. The work of Sams et al. (1999) shows a practical application of the Debeye et al. method to generate multiple 3D realizations of lithology and porosity consistent with geology, petrophysics, and seismic data in a central Sumatra basin mature reservoir. Kane et al. (1999) present a similar geostatistical approach, adding a simple but fast and effective Monte Carlo method to generate solutions. As in the Haas and Dubrule (1994) method, the Debeye et al. (1996), Sams et al. (1999), and Kane et al. (1999) methods as originally published are for inverting a single stacked seismic volume, although the concepts can be extended to multiple partial stack volumes by jointly simulating P- and S-impedance logs at each location.

Francis (2005, 2006a) proposes an alternative geostatistical seismic-inversion method that exploits the advantage of the fast Fourier transform-based spectral simulation to generate impedance realizations, conditional to well data, much faster than sequential simulation techniques. In Francis' method, conditioning to seismic data is accomplished by applying the generalized linear inversion algorithm to update the initial geostatistical realizations of impedance. This method can be used for a joint inversion of multiple seismic volumes, such as near- and far-offset volumes or time-lapse studies, by generating coupled initial conditional realizations. Escobar et al. (2006) have developed a variation of the geostatistical inversion al-

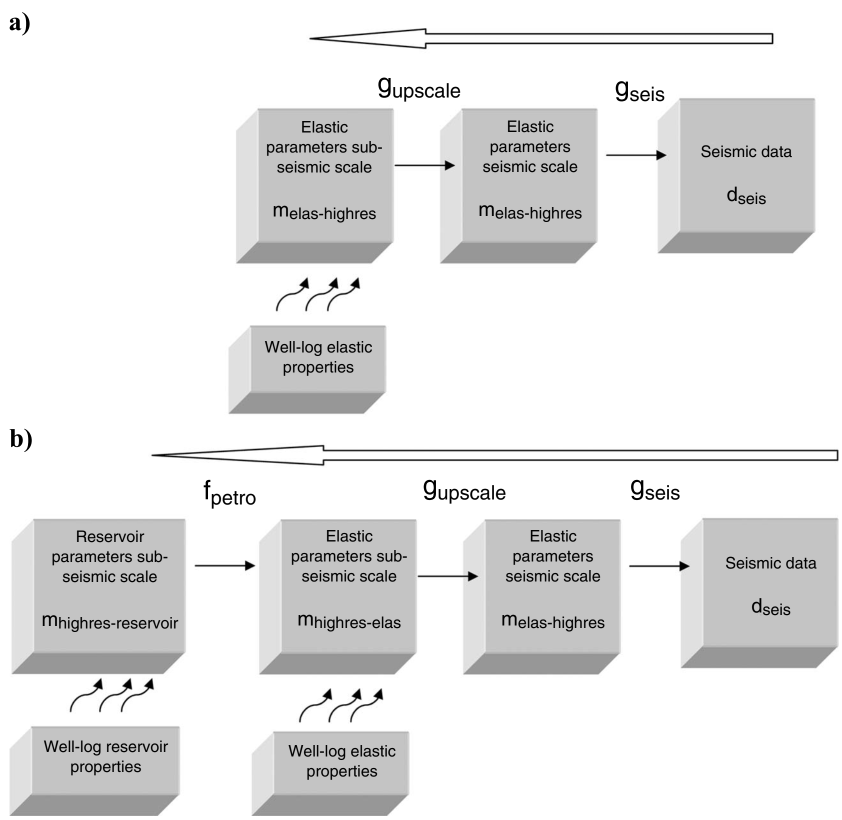

Figure 6. Data and model parameter spaces for the (a) seismic inversion and (b) petrophysical seismic inversion methods conditioned by well-log data. Thin arrows indicate the forward sense; thick arrows indicate the inverse sense; curved arrows indicate conditioning from well-log data. The function $\mathbf{f}_{petro}$ is the rock-physics relation between reservoir parameters and elastic parameters; $\mathbf{g}_{upscale}$ is an upscaling function from subseismic to seismic scale; and $\mathbf{g}_{seis}$ is the forward seismic model that predicts the seismic data from the elastic parameters.

gorithm. By assuming Gaussian priors and likelihoods and by linearizing the forward model, they can approximately decouple the high-dimensional posterior distribution as a product of Gaussian distributions. Then they use sequential Gaussian simulation to draw from the posterior distribution of elastic properties.

Geostatistical seismic inversion schemes have implementation requirements that demand careful consideration. An important one is time-to-depth conversion to relate well-log-based properties to seismic traveltime. A second, equally important consideration is the need to define carefully the directions to drive the lateral correlation. In some geostatistical models, only the vertical variability is characterized and used to constrain the estimated property fields, whereas different traces are considered spatially independent, with the lateral continuity being implicit through the seismic data. The vertical variogram is commonly based on the characterization of the well-log-derived properties.

In more complete formulations, and particularly for conditioning the property fields to well logs, the properties are explicitly modeled as laterally related. The property fields estimated from the seismic inversion or the seismic data can be used to characterize the lateral variability of the strata; other approaches are to adopt the vertical variogram with a range extended with the typical aspect ratio of the strata or an assumed variogram model with a range related to the typical strata correlation length. In general, lateral correlation should be based on a sufficiently fine and well-traced network of horizons, surfaces, or cells that follow the strata. A third issue is the separation of domains by faults, unconformities, and discordances so that spatial correlation is zero across these geologic discontinuities. A thorough integration of well-log data with seismic information requires, in general, a detailed interpreted model.

In the joint work flow discussed earlier, geostatistical seismic inversion can incorporate spatial correlation within the steps of the inversion itself and can be used to draw geologically consistent realizations from the prior $P(\text{facies})$. Then a numerical simulation algorithm such as Markov-chain Monte Carlo can be used to draw realizations from the posterior $P(\text{facies}|\text{attributes})$ using the likelihood $P(\text{attributes}|\text{facies})$. This likelihood involves the rock-physics model relating facies to elastic properties and the wave-propagation forward model relating elastic properties to seismic attributes. Alternatively, under a deterministic approach, a consistent optimal configuration can be estimated to honor within uncertainties the seismic data, the petrophysical relationships, and the spatial model constraints.

Eidsvik et al. (2004) formulate the inversion problem as a simultaneous inversion in terms of a Bayesian network model. They incorporate lateral spatial continuity in the prior distribution of reservoir properties via a Markov random-field model. Larsen et al. (2006) incorporate vertical spatial correlation using a Markov-chain model. Examples of application of the geostatistical inversion method to reservoir characterization include Torres-Verdin et al. (1999) and Contreras et al. (2005).

Figure 7 illustrates in a didactic manner the combination of well-log and seismic data for property estimation in the setting of a petrophysical inversion in the acoustic domain. In this case of a gas reservoir, the seismic data are modeled via the acoustic impedance, total porosity, and water-saturation fields. The impedance is linked to the seismic data by a normal-incidence reflectivity and convolutional model and to the porosity and saturation by a petrophysical model calibrated to well-log data. Conditioning well W1 is at one extreme of the section; well W2 is a control well to validate the results. The first column of the figure shows a geostatistical estimation (kriging) based on well W1's properties and spatial correlation directions that follow a reference reflector in the reservoir. The second column shows a petrophysical inversion with no well-log conditioning. The third column shows the petrophysical inversion conditioned by well W1. The result of the third column capitalizes on information from the two sources, well logs, and seismic data, showing improved vertical resolution and well W2 correlations as compared with the unconditioned inversion.

Many of the geostatistical algorithms used in geostatistical inversion methods rely on two-point statistics (variograms or spatial covariance) to capture geologic continuity. However, the variogram does not incorporate enough information to model complex geologic structures or curvilinear features. To get over this limitation of the two-point geostatistics, Guardiano and Srivastava (1993) present the ideas of training images and multipoint statistics (MPS) for geostatistical simulations. A training image can be defined as a representation of the expected type of geologic variability in the area of study. It reflects the prior geologic knowledge, including the type of features or patterns expected, but it does not need to be conditioned to any hard data. All current MPS algorithms (e.g., Strebelle and Journel, 2001) extract the MPS, i.e., probability of a state at a particular position given the states of multiple neighbors, from a training image.

González et al. (2008) introduce one of the first attempts to use MPS in the seismic-inversion context to obtain reservoir properties directly. Their method combines rock physics and MPS to generate multiple realizations of reservoir facies and saturations, conditioned to seismic and well data. The inversion technique is based on the formulation of the inverse problem as an inference problem, with MPS to characterize the geologic prior information and conditional rock physics to characterize the links between reservoir properties and elastic properties. The González et al. (2008) inversion method provides multiple realizations, all consistent with the expected geology and well-log and seismic data, that honor local rock-physics transformations; yet it does not rigorously sample the solution space or posterior PDF. Their approach combines elements of sampling from conditional probabilities with elements of optimization, providing solutions that limit all possible geostatistical realizations to the ones that can reproduce the available geophysical observations within a certain range of tolerance, given conditional rock-physics distributions.

In González et al.'s (2008) work, rock-physics principles are incorporated at the beginning of the inversion process, establishing the links between reservoir properties (e.g., lithology, saturation) and physical quantities (e.g., impedance, density). It also uses the concept of derived distributions from the statistical rock-physics work flow. Hence, it is possible to predict reservoir conditions not sampled by well-log data, and the consistency between reservoir and elastic properties in solutions is guaranteed. González et al.'s implementation uses a pattern-based MPS algorithm; however, it can be changed by any multipoint geostatistical technique without modifying the core structure of the entire inversion algorithm. Still, there are many ongoing research efforts for developing MPS algorithms capable of handling more realistic training images and for the practical aspects of defining and selecting appropriate training images based on geology.

DISCUSSION

Characterization of reservoir properties by combining seismic inversion, rock physics, and geostatistics depends on the quality of the data, the characteristics of the reservoir, and the methods applied. The elastic properties of rocks are influenced by pore fluids, compaction, cementation, and other factors, and relationships between the lithology, pore fluids, and seismic parameters depend on the specific reservoir setting. In many situations, lithological discrimination is an achievable goal, especially when P- and S-wave attributes are combined within a well-calibrated elastic and rock-physics model. Effects of fluids can cause ambiguities in lithology discrimination. The effects of pore fluids on the elastic properties of the rock depend not only on fluid properties such as density and compressibility but also on the scales of saturations of the pore fluids as well as the pore compliance of the dry rock. Rocks with elastically compliant pore space (e.g., unconsolidated or poorly consolidated sands) are more sensitive to fluid changes than rocks with elastically stiff pores (e.g., well-cemented, well-compacted, unfractured sandstones). Discriminating hydrocarbon from formation water is easier when fluid density and compressibility differences are larger. Hence, shallow reservoirs with elastically compliant rocks and light oil or gas contribute to positive and easy fluid discrimination.

The results of a reservoir-characterization study depend on methods applied. Different workers discuss the main differences between deterministic and stochastic seismic inversion methods (e.g., Helgesen et al., 2000; Sancevero et al., 2005; Sams and Saussus, 2008; Moyen and Doyen, 2009). Francis (2006a, 2006b) presents a particularly nice comparison. In summary, deterministic methods provide a single, local smooth estimate of the subsurface elastic properties with inaccurate assessment of uncertainty. This single, smooth estimate commonly leads to biased estimate of volumes and connectivity. On the other hand, stochastic or Monte Carlo inversion methods provide multiple solutions, all conditional to the seismic data and well observations, allowing better estimates of volumes and connec-

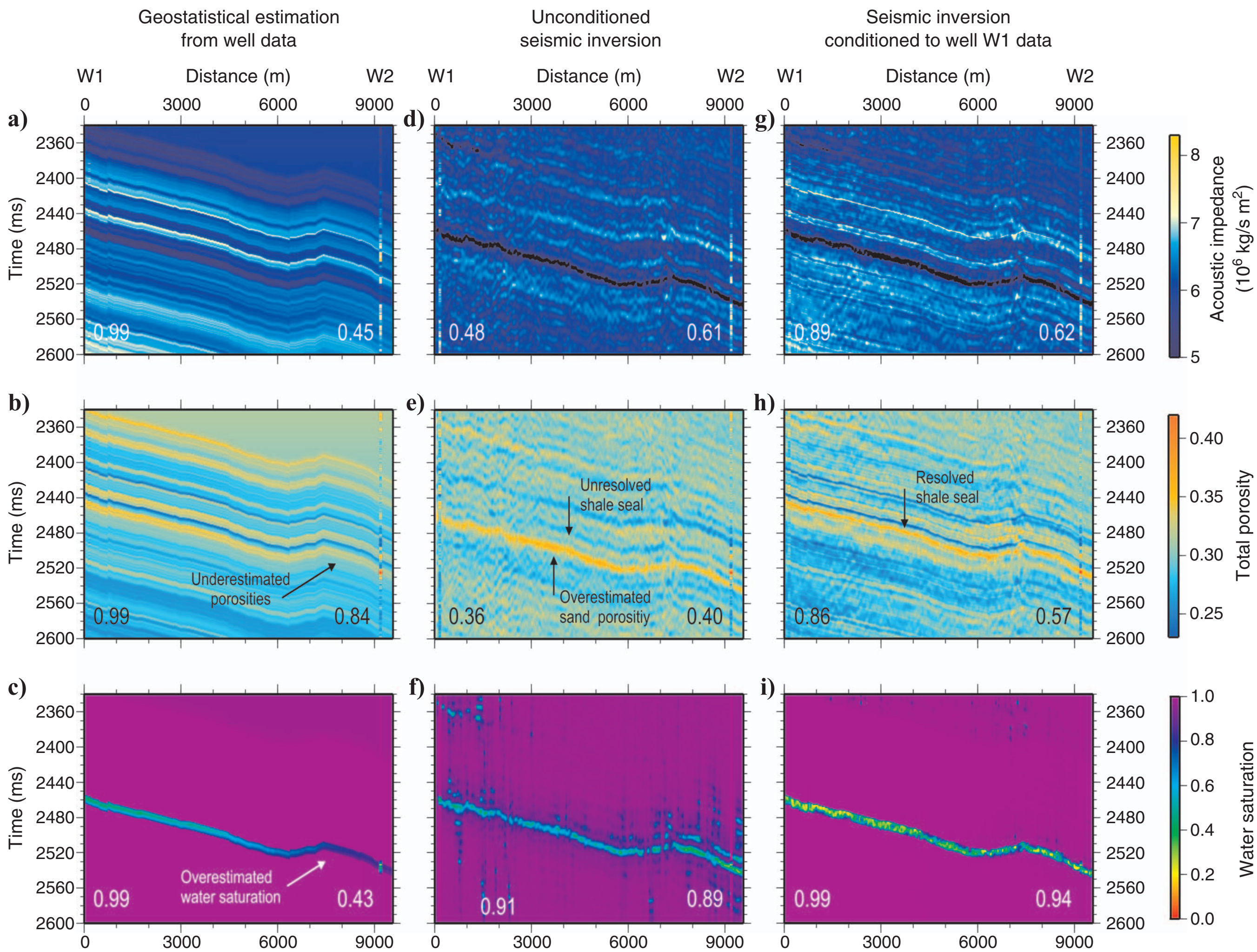

Figure 7. Matrix of plots corresponding to results of the inversion method for (a) acoustic impedance, (b) total porosity, and (c) water saturation. Cokriging of well data is along the structural reference horizon. Middle column is sections estimated by seismic petrophysical inversion with no well-log conditioning for (d) acoustic impedance, (e) total porosity, and (f) water saturation. Right column is sections estimated by the seismic petrophysical inversion conditioned to well W1 data for (g) acoustic impedance, (h) total porosity, and (i) water saturation. At well paths W1 and W2, the well-log-derived properties are superimposed on the corresponding inversion estimates for comparison. Numbers at the bottom of the plots indicate the correlation factor between the estimated property and the corresponding well-log-derived property. Adapted from Bosch et al., 2009a.

tivity along with an appropriate and crucially important uncertainty assessment. Geostatistical inversion for lithofacies, by including realistic geologic spatial models, can better reproduce the shapes of geobodies such as channels and lobes, which are smoothed out in a conventional deterministic seismic inversion (Figure 8).

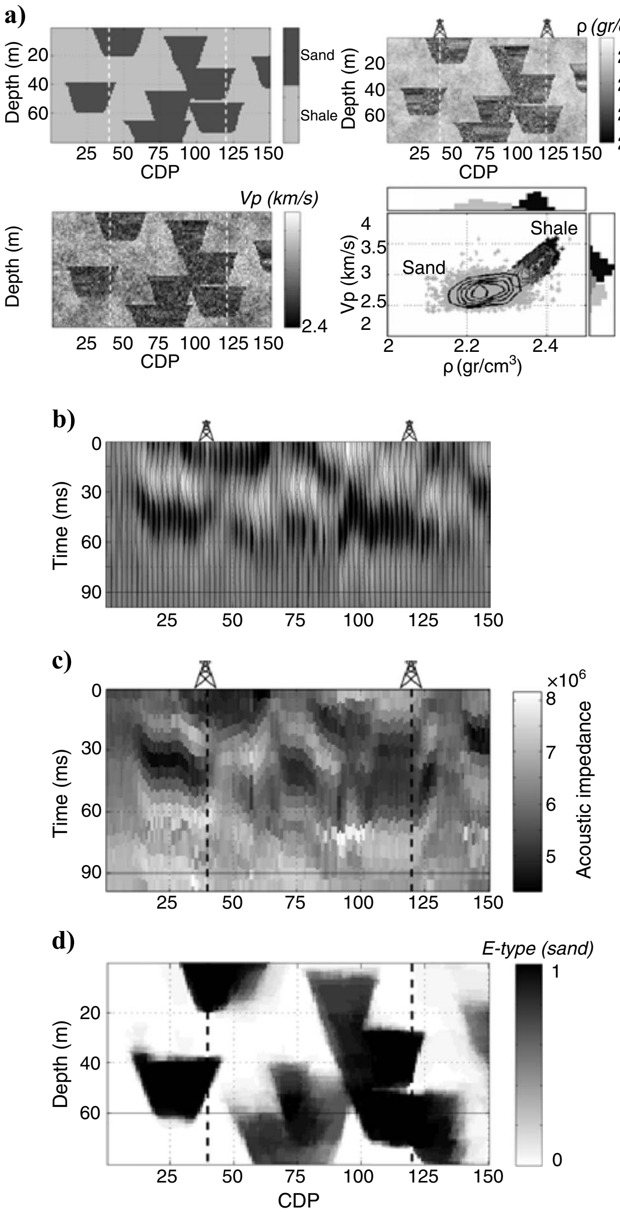

Figure 8. Conventional deterministic inversion compared with inversion incorporating multipoint geostatistics. (a) Input true model, showing spatial distributions of lithology categories (sand and shale), corresponding compressional velocity V_P and density, and V_P-density relation for the two lithologies. (b) Input seismic data for the inversion, computed from the true model using the invariant-embedding reflectivity method (Kennett, 1974) with a 15-Hz Ricker wavelet. (c) Acoustic-impedance section obtained from seismic data in (b) by using a commercially available implementation of the sparse-spike inversion algorithm. (d) Probability (E-type) for sand groups, computed by averaging more than 30 realizations obtained from an MPS-based inversion (González et al., 2008) of the same seismic data. By including geologically realistic spatial models, geostatistical inversion can better reproduce the channels shapes, which are smoothed in conventional deterministic seismic inversion.

Whether deterministic or stochastic, most seismic-inversion work flows require forward seismic modeling. The most common approach has been based on the convolutional model. Although this has been a useful workhorse and has given considerable mileage, we foresee future improvements with wave-equation-based 3D modeling. Presently, CDP modeling is supported by previous prestack seismic migration; it is likely that this process will be accounted for with 3D forward modeling because it is in the general formulation of the full-waveform inversion techniques. Upgrading the isotropic elastic model with more complete mechanical medium models (anisotropic, viscoelastic, poroelastic) would be justified in cases with good-quality data.

Regardless of the potential advantages of Monte Carlo inversion methods, most of the seismic inversion projects use deterministic algorithms (Doyen, 2007). Deterministic inversion methods are easier to apply because of fewer computations involved, ease and availability of software and existing expertise, and therefore less effort and time required to generate results (Sams and Saussus, 2008). On the other hand, partly because of limitations of accessible computer hardware and software in the past, only in recent years have commercial tools and workflows been available to overcome the difficulties of handling and interpreting the multiple realizations of any geostatistical inversion scheme.

Additionally, as Doyen (2007) points out, there have been some misconceptions about the resolution of the results obtained with geostatistical inversion methods. Monte Carlo inversions can provide solutions at any desired sampling; however, that does not mean the seismic resolution is increased. Only within the range of well-log information is the vertical resolution of the model (not the seismic data) increased by spatial conditioning of the model properties to the fine-scale, log-derived properties. In geostatistical inversion methods, the temporal frequency content outside the seismic bandwidth is simulated, in the geostatistical sense, based on the spatial correlation model selected and maintaining consistency with well observations. In fact, as suggested, the way geostatistical inversion assesses uncertainties resulting from the band-limited nature of the seismic data is by generating multiple full-bandwidth impedance or elastic-property models consistent with the seismic data, spatial correlations, and well observations.

Finally, a word of caution about using multiple geostatistical realizations as a measure of uncertainty (Dubrule et al., 1996; Dubrule, 2003). It has been argued (Massonnat, 2000) that uncertainties associated with different geologic scenarios are far more significant than those captured by multiple realizations under a single scenario. Different scenarios obtained from different plausible geologic concepts about the depositional system must be combined with geostatistical simulations for each scenario. Lia et al. (1997), Corre et al. (2000), Charles et al. (2001), and Dubrule and Damsleth (2001), among others, discuss approaches for quantifying uncertainty in reservoir characterization by combining geologic scenarios with geostatistical realizations. Some future directions of research could include stronger coupling of geology with quantitative reservoir models, use of geostatistical inversion for prestack seismic data, and tighter integration of seismic reservoir characterization with matching production history and making decisions related to reservoir management. Assessing uncertainty in and of itself may have little value unless it is linked to specific reservoir decisions.

CONCLUSION

We have reviewed seismic-inversion schemes (deterministic and stochastic) that incorporate rock-physics information and geostatistical models of spatial continuity. Seismic inversion by itself is a successful technique to predict elastic properties from seismic data. However, reservoir characterization requires techniques that go beyond inverting for elastic parameters (e.g., impedances, elastic moduli) and that try to infer reservoir properties of interest, such as lithology, porosity, and fluid saturation. Empirical or theoretical statistical rock physics provides information serving as the bridge between elastic and reservoir properties, whereas geostatistical methods impose on the estimated or simulated property fields the appropriate spatial variability and coherence with localized information, typically from well logs. The most general formulation is given in terms of Bayesian inference, i.e., modeling information by probability densities in parameter spaces and integrating the information to produce a combined posterior probability density.

The inference problem can be separated into its information components — seismic data, rock physics, and geostatistics — and linked via various cascaded work flows or formulated jointly. Once the formulation and relevant parameters are chosen according to the case, the solution methods can be grouped into two major approaches: optimization and sampling. The first one uses algorithms that search for the maximum of the combined probability, commonly at a local mode; the second uses techniques that produce an ensemble of samples (realizations) from the combined probability density to provide marginal parameter estimates and probabilistic interpretations. The technique, in various possible work flows, is a major contribution to reservoir characterization and should be considered part of an almost standard and essential work flow after seismic processing.

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