Joint gravity and magnetic inversion in 3D using Monte Carlo methods

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ABSTRACT

We jointly invert gravity and magnetic data following a Monte Carlo method that provides estimation for a 3D model of the structure and physical properties of the medium. In particular, layer interface depths, the density and magnetic susceptibility fields within layers are estimated, and their uncertainties are described with posterior probabilities. This method combines the gravity and magnetic data with prior information on the mass density, magnetic susceptibility statistics, and statistical constraints on the interface positions. The resulting model realizations jointly comply with the observations and the prior statistical information.

INTRODUCTION

Hydrocarbon reservoirs in many areas are several km deep and are related with complex geologic and tectonic processes. Images obtained from seismic surveys maybe unable to show the deep structure of the sedimentary cover and the crystalline basement. Thus, gravity and magnetic field data are important in these areas to estimate the basement geometry and the large-scale structure of the basins.

Certainty in the estimation based on potential field data (gravity and magnetic data) is commonly affected by the non-unique relationship between model and data spaces, i.e. a set of model configurations equally explain the data. In addition, other sources of uncertainty like observation and modeling errors are involved. To improve the inference and reduce uncertainty we combine the available information in the area: gravity and magnetic data, statistics for the density and magnetic susceptibility and constraints on mayor sedimentary structure.

Our approach is based on a statistical formulation for joint inversion of multiple geophysical data (Bosch, 1999) previously applied to joint inversion of gravity and magnetic data in 2D (Bosch et al, 2001; Bosch and McGaughey, 2001). This method allows for quantitative integration of gravity, magnetic, petrophysical and other prior information, to produce an estimation of the major layer structure geometry and the property fields inside the layers. The formulation is solved using Monte Carlo methods (Mosegaard and Tarantola, 1997) providing as result a description of the uncertainties via the assessment of model parameter probabilities

THEORY AND METHOD

We formulate the inverse problem with a statistical approach and describe each type of information with probability density functions (*pdf*s) on a model parameter space. The posterior probability density function combines the different components of the information and is given by (Bosch, 1999),

$$\sigma(\mathbf{m}) = \text{const } \rho(\mathbf{m}) \ L_{grav}(\mathbf{m}) \ L_{mag}(\mathbf{m}), \qquad (1)$$

where $\mathbf{m}$ is the array of model parameters describing the structure and medium properties. In the equation above the posterior density function, $\sigma(\mathbf{m})$, is a product of three factors: the prior probability density function, $\rho(\mathbf{m})$, and the likelihood functions, $L_{grav}(\mathbf{m})$ and $L_{mag}(\mathbf{m})$, associated with the gravity and magnetic data correspondingly.
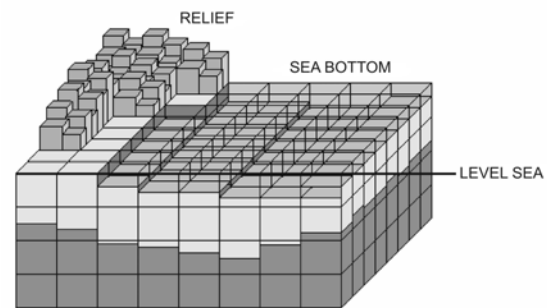

Figure 1. Model parameterization showing sediment and basement layers, cells used to describe the mass density and magnetic susceptibility 3-dimensional fields, the topographic relief and the sea bottom.

In the present case, the model space is a composition of three subspaces,

$$\mathbf{m} = (\mathbf{m}_z , \mathbf{m}_{den} , \mathbf{m}_{sus}) , \qquad (2)$$

where $\mathbf{m}_z$ indicate the parameters defining the geometry of the model layers, $\mathbf{m}_{den}$ and $\mathbf{m}_{sus}$ are the parameters defining the mass density and magnetic susceptibility fields respectively. The prior probability density is formulated as,

$$\rho(\mathbf{m}) = \text{const } \phi (\mathbf{m}_{den} , \mathbf{m}_{sus} | \ \mathbf{m}_z) \ \rho_z(\mathbf{m}_z), \qquad (3)$$

the product of a marginal probability density and a conditional probability density. The former, is the prior *pdf* on the layer structure $\rho_z(\mathbf{m}_z)$, containing the information on the positions and geometry of each layer interface represented in the model. The later, $\phi(\mathbf{m}_{den}, \mathbf{m}_{sus} | \mathbf{m}_z)$, contains information on the statistical distribution of the density and the magnetic susceptibility within each layer. These probability densities are modeled with a multivariate Gaussian function that takes into account the properties spatial covariance and the correlation between density and magnetic susceptibility.

Table 1. Parameters defining the statistical model for medium properties and basement interface.

PARAMETER	SEDIMENT	BASEMENT	UNIT
Mean density	2136	2850	kg/m3
Mean susceptibility	-6	-2	Log_{10} (SI)
Standard deviation for density	100	100	kg/m3
Standard deviation for susceptibility	0.8	0.8	Log_{10} (SI)
Covariance range for properties in X direction	50	50	km
Covariance range for properties in Y direction	50	50	km
Covariance range for properties in Z direction	10	10	km
Covariance range for interface depth in X direction	----	100	km
Covariance range for interface depth in Y direction	----	50	km
Standard deviation for interface depth	----	1	km

The solution of the inverse problem is obtained using a Markov Chain Monte Carlo method adapted to sample model realizations according to the posterior density described in equation (1). As a result, the model realizations generated by the chain honor the different types of information combined: gravity and magnetic observations, statistical information on density and susceptibility, and statistical constraints on the prior configuration of the interfaces.

NUMERICAL EXAMPLE

We define in the area a geological model for a sedimentary basin with three layers: (1) a water body describing the sea volume, (2) a sedimentary layer, and (3) the crystalline basement layer. The topography is used to describe the mountain relief and is taken into account to calculate the gravity and magnetic fields, and the bathymetry is used to describe the seabed geometry.

We parameterize the model describing the interface that separate the layers, defining the depth interface values on a regular 2D grid, and the values of density and magnetic

susceptibility in regular blocks within each layer, as shown in Figure 1.

We created a synthetic model to test the inversion method, using true bathymetry, topography data and interpreted basement depths from a northwestern region of Venezuela. From the "true" model we calculated the corresponding "observed" gravity and "magnetic" data to be used in the inversion test. The statistical parameters used to define the property and interface statistical prior model are described in Table 1.

INVERSION RESULTS

All model parameters, with the exception of the bathymetry and topography, were modified with the Monte Carlo inversion algorithm to fit the data, explore the model space and generate the realizations from the posterior probability density. We used a combination of Gibbs and Metropolis samplers in the algorithm to produce a chain of 2.5 million iterations. A local modification of model parameters is proposed per iteration: variation of an interface depth at a grid node or the physical properties (density and susceptibility) of a cell in the model. The parameter modifications are accepted according to the posterior probability, equation 1. Hence, the models generated by the chain jointly comply with the geological model, the prior statistics of physical properties and interfaces and the gravity and magnetic observations.
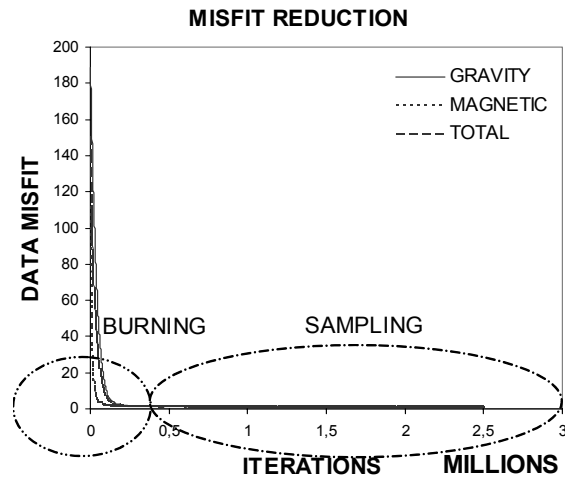

Figure 2. Progress in misfit reduction with iterations of the sampling algorithm.

Figure 2 shows the progress in reduction of data misfit as iterations proceed. As usually in Monte Carlo methods, the first part of the chain (burn-in period) is influenced by the initial model and will not be used for posterior statistics. The phase of convergence, which is indicated with the reduction of the data misfit, involved the first 0.5 million iterations. Hence, we generated 2 million model configurations to calculate the parameter estimates and the posterior probabilities.

Figure 3 shows the "true" basement depth and the correspondent "observed" gravity and magnetic data in the area, as well as the basement depth, calculated gravity and calculated magnetic data form a model in the chain taken at random.

FREE AIR GRAVITY ANOMALY (mGal)

OBSERVED GRAVITY (mGal)

CALCULATED GRAVITY (mGal)
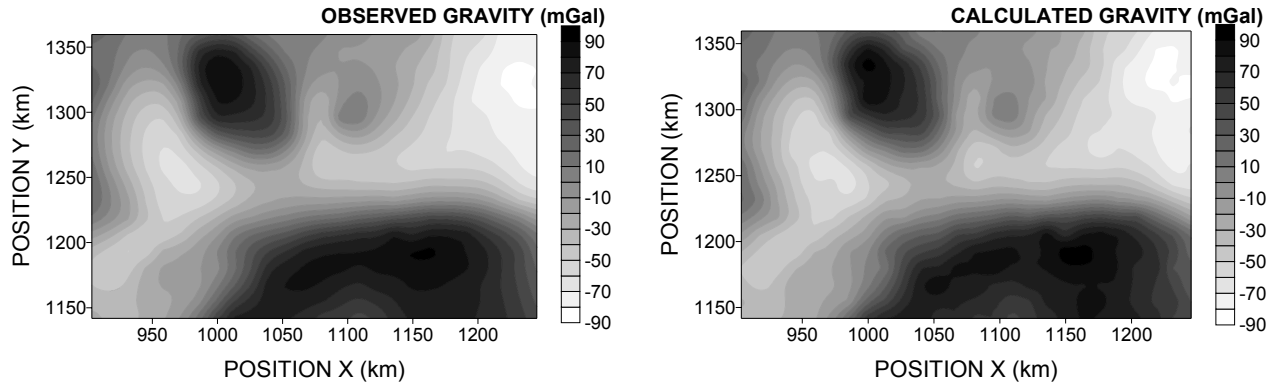
TOTAL MAGNETIC INTENSITY ANOMALY (nT)

OBSERVED MAGNETIC (nT)

CALCULATED MAGNETIC (nT)

BASEMENT DEPTH (km)

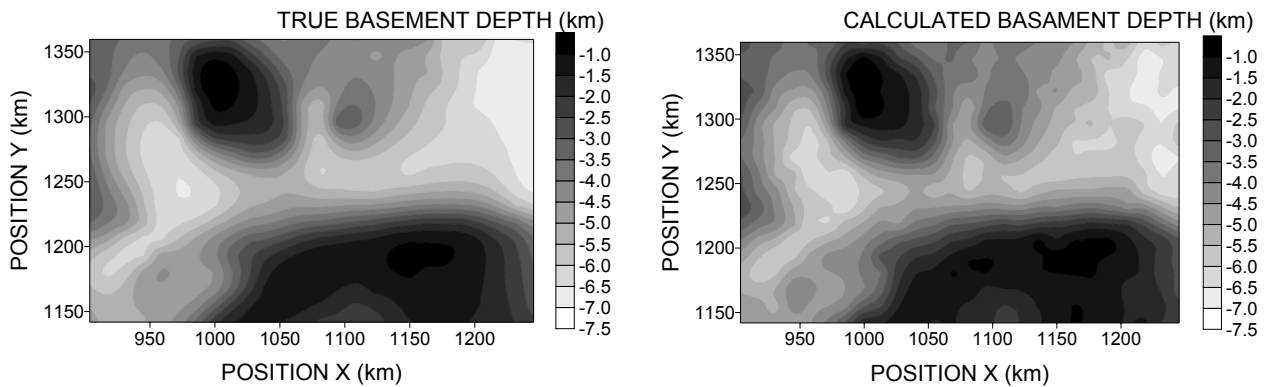### TRUE BASEMENT DEPTH (km)
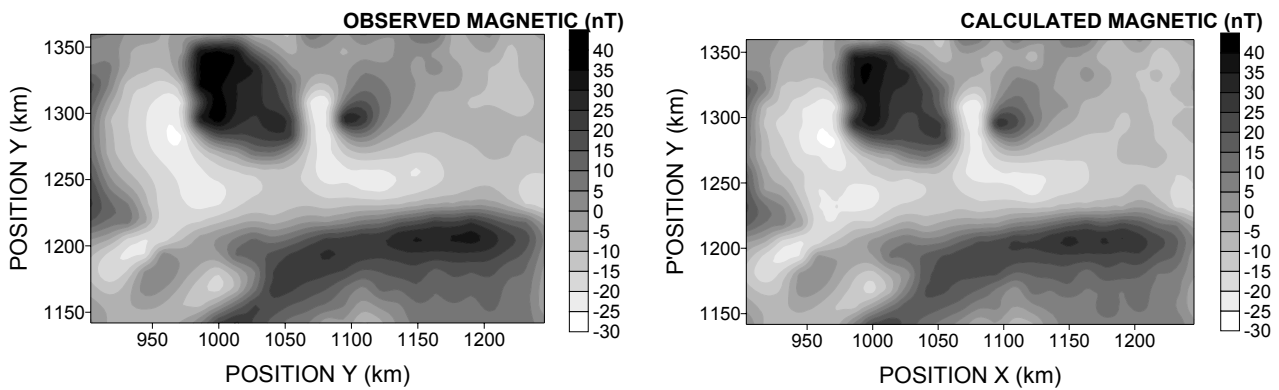
CALCULATED BASAMENT DEPTH (km)

Figure 3. "True" basement interface with the corresponding "observed" gravity and magnetic data, and a realization sampled from the posterior probability density function using the Monte Carlo method with the corresponding calculated gravity and magnetic data.

We used for the likelihood function term, in equation 1, a data uncertainty of 2% of the range of the anomaly. The figure shows the adequate fit between the observed and calculated anomalies.

From the chain of model realizations we calculate for each layer a volume of probabilities for the occurrence of the layer in given coordinate positions. The probabilities correspond to the frequency of the layer in the given position divided by the number of realizations. Figure 4b show a probability plot at a vertical section through this volume. Figures 4c and 4d show plot examples of probability for the depth to the basement, bellow given coordinate locations in the surface. The plots fully describe the uncertainty in the location of the geological bodies included in the model.

DEPTH (km)

-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
-4.0
-4.5
-5.0
-5.5
-6.0
-6.5
-7.0
-7.5

B

PROBABILITY

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

C

LOCATION (1000, 1200)

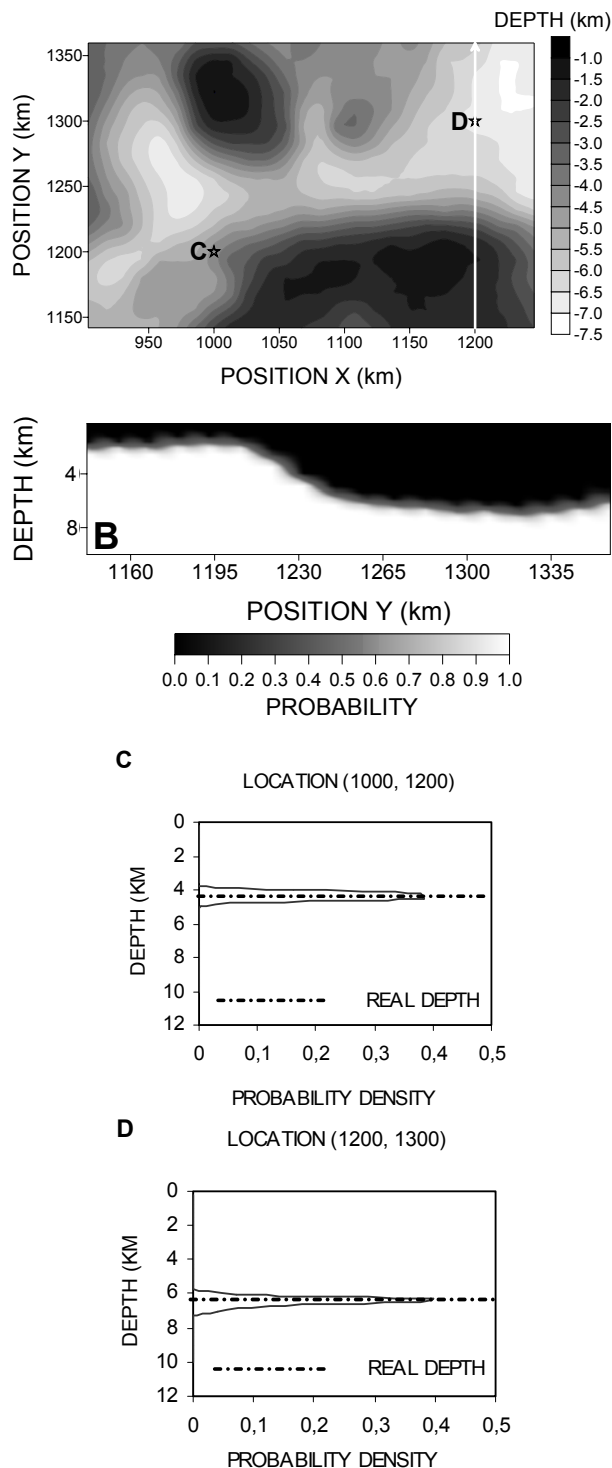REAL DEPTH

D

LOCATION (1200, 1300)

REAL DEPTH

Figure 4. Probability plots summarizing inversion results. (a) True depths to crystalline basement in the area. (b) Vertical section showing probability for finding the crystalline basement rocks. Location of the trace of the section is shown with a white arrow in plot 4a. The two white stars in plot 4a indicate location of the two vertical probability density plots, (c) and (d), which show probability of depth for the sedimentary basin basement. The grey dashed line indicates true basement depth.

CONCLUSION

The Monte Carlo inversion method used in this work allows us to successfully combine in a quantitative way different types of information in 3D, estimating model parameters and describing their posterior probabilities. By combining gravity and magnetic data with a statistical model for the physical medium properties and interface geometry we infer the structure of the sedimentary basin in a numerical example, jointly explaining the two observed potential fields and honouring the prior statistical model.

ACKNOWLEDGEMENTS

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